

# From Nonlinear Integrated Optics to Microresonator Frequency Combs

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Perhaps one of the most spectacular current applications of nonlinear integrated optics, a field which was pioneered by George Stegeman more than thirty years ago [1], is that of nonlinear microresonator based optical frequency comb light sources. Optical frequency comb sources are characterized by a spectrum comprising many equally spaced components [2], and have a wide range of scientific and technological applications. Although commercial comb generators are based on mode-locked lasers and fiber supercontinuum generation, nonlinear integrated optics provides a low-cost and chip-scale alternative, based on a low-power cw laser coupled into a high-Q microresonator [3]. So far microresonator frequency combs have exploited the third order “Kerr” nonlinearity, which permits to generate successive comb lines with a spacing equal to the resonator free-spectral range via cascaded four-wave mixing [4-5]. Modeling of microresonator frequency combs can be greatly simplified by a single partial differential equation approach [4-6], analogous to the case of other coherently driven Kerr spatially diffractive [7] or temporally dispersive [8-9] nonlinear cavities. In order to lower the threshold power and extend the spectral range of frequency comb generation, for example into the visible or mid-infrared, while still using near-infrared cw laser pumps, quadratic nonlinear cavities can be exploited [10]. These quadratic microresonator frequency comb sources operate close to the phase-matching condition for the underlying quadratic processes, and not in the cascading regime that reduces the dynamics to the Kerr case [11]. Quite remarkably, a single time domain partial differential equation with an effective delayed third-order nonlinearity was derived to describe with excellent accuracy the dynamics of quadratic frequency comb generation [12-13]. In situations where multiple processes are present, and the frequency combs generated around the interacting waves over multiple octaves overlap, we carried out numerical modeling based on a single envelope equation approach [14].

## References

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